Ledgers for Math and Science

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with use‡ of Claude (Anthropic)

Foreward

Families often approach me when math and physics students have repeatedly struggled on tests despite completing all school homework and being told to “be careful”. *After* tests are handed back and students are guided through solutions, the subtle steps they missed become clear. But how can one know how to “be careful” *before* the next test?

Read a good textbook (even if textbook reading isn’t assigned) and move reasoning out of your head and into organized written work.

Take care of both these tasks using this packet, which demonstrates how to document thinking in tables to make technical cognition a deliberate craft exposed to adversarial scrutiny, rather than intuition that either arrives or doesn’t (and then snowballs into desperate guesses and the disintegration of mental self-control).

Beyond improving academic performance, learning to be careful with tables is meant to give students a capacity to acquire and apply unfamiliar material without supervision. Ideally, this capacity permits students to become forward-deployed/context/knowledge engineers, hardware/software engineers, lawyers, principal pharmaceutical researchers, quantitative analysts, or similarly compensated analysts in careers of their own creation.

‡ I, David Liao, used Claude as a research and drafting assistant. I independently verified all text, factual claims, and cited sources against the primary sources. I am solely responsible for the final content.

See pg. 16 for exercises.

Example 1.1: Solve for x.Show each step that can't be accomplished on a basic four-function calculator.

$$4x - 2 = 98$$

	Statement	**Tool**	**Equivalent parts** (of statement & tool)	**Populated tool / Point(s)**
1.	$4x - 2 = 98$	$a = b$ $\updownarrow$ $a + c = b + c$	$a = 4x - 2$ $b = 98$ $c = 2$	$4x - 2 = 98$ $\updownarrow$ $4x - 2 + 2 = 98 + 2$
2.	$4x + 0 = 100$	$a \leftrightarrow a + 0 \leftrightarrow 0 + a$	$a = 4x$	$4x \leftrightarrow 4x + 0 \leftrightarrow 0 + 4x$
3.	$4x = 100$	$a = b$ $\updownarrow$ $\frac{a}{c} = \frac{b}{c}, c \neq 0$	$a = 4x$ $b = 100$ $c = 4$	$4x = 100$ $\updownarrow$ $\frac{4x}{4} = \frac{100}{4}, 4 \neq 0$
4.	$\frac{4x}{4} = 25$	$a \leftrightarrow a \cdot 1 \leftrightarrow 1 \cdot a$	$a = 4$	$4 \leftrightarrow 4 \cdot 1 \leftrightarrow 1 \cdot 4$
5.	$\frac{4x}{4 \cdot 1} = 25$	$\frac{ab}{cd} \leftrightarrow \frac{a}{c}\frac{b}{d}$	$a = 4$ $b = x$ $c = 4$ $d = 1$	$\frac{4x}{4 \cdot 1} \leftrightarrow \frac{4}{4}\frac{x}{1}$
6.	$\frac{4}{4}\frac{x}{1} = 25$	$\frac{a}{a}, a \neq 0 \leftrightarrow 1$	$a = 4$	$\frac{4}{4}, 4 \neq 0 \leftrightarrow 1$
7.	$1 \cdot \frac{x}{1} = 25$	$\frac{a}{1} \leftrightarrow a$	$a = x$	$\frac{x}{1} \leftrightarrow x$
8.	$1 \cdot x = 25$	$a \cdot 1 \leftrightarrow 1 \cdot a \leftrightarrow a$	$a = x$	$x \cdot 1 \leftrightarrow 1 \cdot x \leftrightarrow x$
9.	$x = 25$			

To fill in the **Tool** column, use the basic algebra tools or basic pre-algebra tools on the following pages.

To obtain a two-column Statement-Reason proof in geometry, keep only the **Statement** and **Tool** columns of a STEP table, move the entries in the **Tool** column down one row, and write "Given" (typically) in the newly vacant entry at the top of the **Tool** column. Homework and tests in Algebra 1 and 2 classes typically only require the work in the **Statement** column to be shown.

Example 1.2: Explain your reasoning for row 1 in the STEP table from Example 1.1.

<table>
<tr><th></th><th>Style</th><th>Skip statement</th><th>Tool</th><th>Equivalent parts (of statement & tool)</th><th>Populated tool / Point(s)</th></tr>
<tr><td>1.</td><td>Algebraic</td><td rowspan="4">When asked to write an explanation, you often need not rewrite the problem statement or current line of work, which is typically already nearby.</td><td>By the Addition Property of Equality, $a = b$ implies $a + c = b + c$.</td><td>Regard $4x - 2$ as a, 98 as b, and 2 as c.</td><td>So, $4x - 2 = 98$ implies $4x - 2 + 2 = 98 + 2$.</td></tr>
<tr><td>2.</td><td>Natural</td><td>By the Addition Property of Equality, a common quantity can be added to both sides of a true equation.</td><td>So, in the equation setting left side $4x - 2$ equal to right side 98, adding 2 to both sides</td><td>gives $4x - 2 + 2 = 98 + 2$.</td></tr>
<tr><td>3.</td><td>Brief (merged parts)</td><td colspan="3">By the Addition Property of Equality, a common quantity, say 2, can be added to both sides of a true equation, say $4x - 2 = 98$. So $4x - 2 + 2 = 98 + 2$.</td></tr>
<tr><td>4.</td><td>Even shorter (omit name of tool)</td><td colspan="3">A common quantity, say 2, can be added to both sides of a true equation, say $4x - 2 = 98$. So $4x - 2 + 2 = 98 + 2$.</td></tr>
</table>

Basic Algebra Tools

For exercises, see corresponding "EA" sections in Marecek *et al.*, *Elementary Algebra 2e*, available for free through a CC BY license at OpenStax: openstax.org/details/books/elementary-algebra-2e

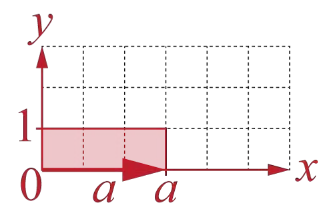
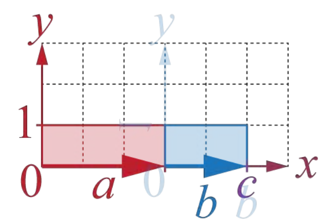
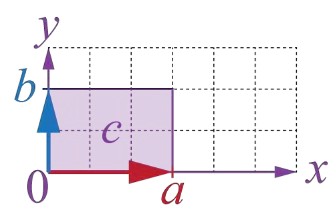
EA	Name	Tool(s)
1.9	**A.** Division by zero is undefined	Any denominator or factor in denominator = 0 ↓ Discard template
1.5	**B.** Product of fractions	$\frac{a}{b}\cdot\frac{c}{d}\leftrightarrow\frac{ac}{bd}$
1.5	**C.** Canceling factors in numerator & denominator	$\frac{a\cdot c}{b\cdot c}\leftrightarrow\frac{a}{b}$ $\frac{c\cdot a}{c\cdot b}\leftrightarrow\frac{a}{b}$
1.5	**D.** Quotient of fractions	$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}\leftrightarrow\frac{a}{b}\cdot\frac{d}{c}$
1.9	**E.** Commutative Property of Addition	$a+b$ $\updownarrow$ $b+a$
1.9	**F.** Commutative Property of Multiplication	$a\cdot b$ $\updownarrow$ $b\cdot a$
1.9	**G.** Associative Property of Addition	$a+b+c$ $\updownarrow$ $(a+b)+c$ $\updownarrow$ $a+(b+c)$
1.9	**H.** Associative Property of Multiplication	$a\cdot b\cdot c$ $\updownarrow$ $(a\cdot b)\cdot c$ $\updownarrow$ $a\cdot(b\cdot c)$

EA	Name	Tool(s)
1.9	**I.** Distributive Property	$a\cdot(b+c)$ $\updownarrow$ $a\cdot b+a\cdot c$ $(b+c)\cdot a$ $\updownarrow$ $b\cdot a+c\cdot a$ $a\cdot(b-c)$ $\updownarrow$ $a\cdot b-a\cdot c$ $(b-c)\cdot a$ $\updownarrow$ $b\cdot a-c\cdot a$ $(a+b)\cdot(c+d)$ $\updownarrow$ $\begin{array}{c\|c\|c} & c & d \\ \hline a & a\cdot c & a\cdot d \\ \hline b & b\cdot c & b\cdot d \end{array}$ $a\cdot c+b\cdot c+a\cdot d+b\cdot d$
1.9	**J.** Identity Property of Addition	$a+0\leftrightarrow 0+a\leftrightarrow a$
1.9	**K.** Identity Property of Multiplication	$a\cdot 1\leftrightarrow 1\cdot a\leftrightarrow a$
1.9	**L.** Identity Property of Division	$\frac{a}{1}\leftrightarrow a$
1.9	**M.** Adding additive inverse is equivalent to subtracting	$a+(-b)$ $\updownarrow$ $a-b$
1.9	**N.** Inverse Property of Addition	$a+(-a)\leftrightarrow a-a\leftrightarrow 0$
1.5 & 1.9	**O.** Inverse Property of Multiplication (and the related One Property of Division)	$a\cdot\frac{1}{a}, a\neq 0\leftrightarrow\frac{a}{a}, a\neq 0\leftrightarrow 1$

EA	Name	Tool(s)	
1.9	**P.** Multiplication by zero	$a \cdot 0 \leftrightarrow 0 \cdot a \leftrightarrow 0$	
1.9	**Q.** Division involving zero	$\frac{0}{a} \leftrightarrow 0$	See "Division by zero is undefined"
2.1	**R.** Addition Property of Equality	$a = b$ $\downarrow$ $\begin{array}{ccc} a & = & b \\ +c & & +c \\ \hline a+c & = & b+c \end{array}$	
2.1	**S.** Subtraction Property of Equality	$a = b$ $\downarrow$ $\begin{array}{ccc} a & = & b \\ -c & & -c \\ \hline a-c & = & b-c \end{array}$	
2.2	**T.** Multiplication Property of Equality	$a = b$ $\downarrow$ $a \cdot c = b \cdot c$	
2.2	**U.** Division Property of Equality	$a = b$ $\downarrow$ $\frac{a}{c} = \frac{b}{c}$	

EA	Name	Tool(s)	
6.2	**V.** Exponent notation	$a^2 \leftrightarrow a \cdot a$	$a^n \leftrightarrow \underbrace{a \cdot a \cdots a}_{n \text{ copies}}$
7.6	**W.** Zero-Product Property	$a \cdot b = 0$ $\downarrow$ $a = 0$ or $b = 0$	
9.1/9.7	**X.** Root notation	$\sqrt[n]{a} = b$, even n $\updownarrow$ $b^n = a, b \geq 0$, even n	$\sqrt[n]{a} = b$, odd n $\updownarrow$ $b^n = a$, odd n
9.8*	**Y.** A correction to the Power Property	$\sqrt{a^2} \leftrightarrow \lvert a \rvert$	
10.3	**Z.** Quadratic Formula	$ax^2 + bx + c = 0$ $\updownarrow$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

BASIC PRE-ALGEBRA TOOLS

	Name	Numeric example	Tool: Algebraic	Tool: Gridded	Gridding steps
A.	**(Real) number**	3	a		1. Draw horizontal arrow from origin (0) to x-tick labeled a. Label arrow a. 2. Regard horizontal arrow as bottom edge of a rectangle of height 1. 3. Horizontal arrow and rectangle represent the number a individually and together.
B.	**Addition**	$3 + 2 = 5$	$a + b = c$		1. Represent the number a. 2. Regard arrowhead of arrow for a as origin of a new axis system. 3. Use new axis system to represent the number b. 4. Use original axes to read off x-tick, c, of arrowhead you just drew and complete sketch of the number c.
C.	**Subtraction**	$2 = 5 - 3$	$b = c - a$		
E.	**Multiplication**	$(3)(2) = 6$	$(a)(b) = c$		1. Represent number a. Erase rectangle. 2. Draw vertical arrow from origin (0) to y-tick labeled b. 3. Draw rectangle using horizontal arrow for a and vertical arrow for b as sides. Label signed number of unit squares, c, rectangle covers. 4. A rectangle in Quadrant I or Quadrant III has a positive signed number of unit squares. A rectangle in Quadrant II or Quadrant IV has a negative signed number of unit squares.
F.	**Division**	$2 = \frac{6}{3}$	$b = \frac{c}{a}, a \neq 0$		

Example 2.1: Last year, Cities A and B had median SAT scores of 1200 and 1350, respectively. Charlie's model proposes that a higher city-wide median SAT score last year means a higher city-wide median home price. According to Charlie's model, which city has a higher median home price?

<table>
<tr><th></th><th colspan="3">Statement</th><th colspan="3">Tool</th><th colspan="3">Equivalent parts
(of statement & tool)</th><th colspan="3">Populated tool / Point(s)</th></tr>
<tr><td rowspan="3">1.</td><td colspan="3" rowspan="3">(Restating a word problem is not usually required).</td><td colspan="3">Charlie's model</td><td>City A
median SAT score 1200</td><td></td><td>City B
median SAT score 1350</td><td>City A
median SAT score 1200</td><td></td><td>City B
median SAT score 1350</td></tr>
<tr><td>lower median SAT score</td><td></td><td>HIGHER median SAT score</td><td>lower median SAT score</td><td></td><td>HIGHER median SAT score</td><td>lower median SAT score</td><td></td><td>HIGHER median SAT score</td></tr>
<tr><td>lower median home price</td><td></td><td>HIGHER median home price</td><td></td><td></td><td></td><td>lower median home price</td><td></td><td>HIGHER median home price</td></tr>
<tr><td rowspan="2">2.</td><td>City A
median SAT score 1200</td><td></td><td>City B
median SAT score 1350</td><td colspan="3" rowspan="2"></td><td colspan="3" rowspan="2"></td><td colspan="3" rowspan="2"></td></tr>
<tr><td>lower median home price</td><td></td><td>HIGHER median home price</td></tr>
</table>

According to Charlie's model, City B has a higher median home price.

In fast work, the charts in **Tool**, **Equivalent parts**, and **Populated tool / Point(s)** overlap in the same workspace (no need to create three separate charts).

Example 2.2: Explain your reasoning for Example 2.1.

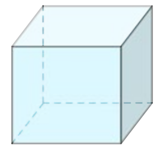
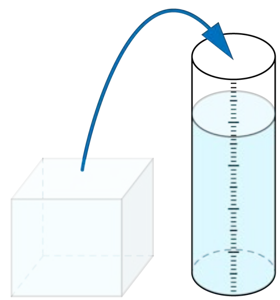
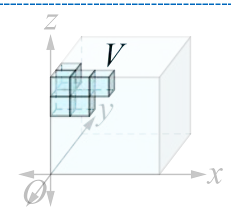
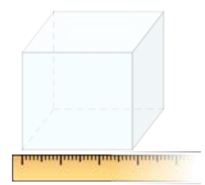
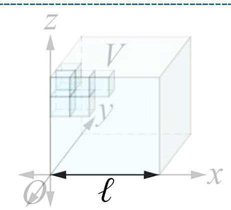
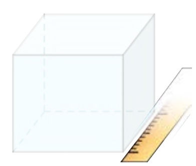
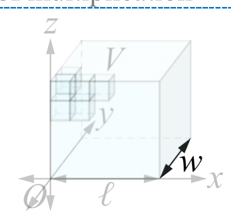
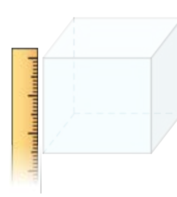
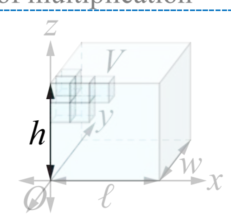
	Skip statement	**Tool**	**Equivalent parts** (of statement & tool)	**Populated tool / Point(s)**
1.	(Restating a word problem is not usually required).	By Charlie's model, higher median SAT score means higher median home price.	City B's median SAT score of 1350 was higher than City A's median SAT score of 1200.	So, by Charlie's model, City B has a higher median home price.
2.	When the **Populated tool / Point(s)** entry in the previous row gives just the concise conclusion and answers the question, an additional row restating the conclusion is not needed.			

Example 2.3: Briefly explain your reasoning for Example 2.1.

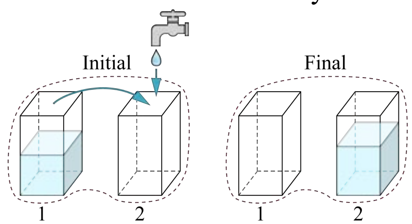
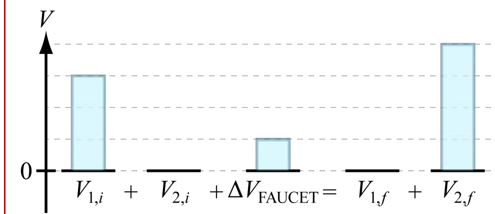
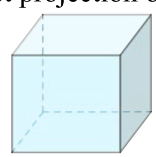
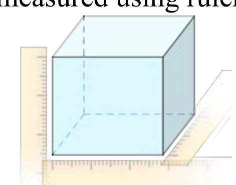
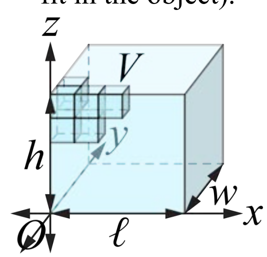
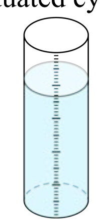
By Charlie's model, City B has a higher median home price because City B's median SAT score of 1350 was higher.

Language-craft procedure:
1. Omit the **Tool** entry.
2. Reverse the order of the **Populated tool / Point(s)** and **Equivalent parts**.
3. Join the clauses into a single sentence using "because".
4. Omit the "So".

Example 3.1: Translate: "A rectangular parcel of water has a volume equaling the parcel's length times the parcel's width times the parcel's height."

	Short phrase	Cartoon	Axes and plots	Notation
1.	A rectangular parcel of water	Semi-transparent rectangular prism 		
2.	has a volume	Arrow indicating water can be poured from rectangular container into graduated cylinder used to measure volume 	Cartesian axis system implying scale for representative unit cubes within parcel 	Let V = Volume
3.	equaling		Optional: Number line with two arrows pointing from the origin to same tickmark	$V =$
4.	the parcel's length	Optional: Ruler used to measure length 	Double-headed arrow indicating size of an edge parallel to x-axis 	Let ℓ =width $V = \ell$
5.	times		Optional: Area model of multiplication	$V = \ell \cdot$
6.	the parcel's width	Optional: Ruler used to measure width 	Double-headed arrow indicating size of an edge parallel to y-axis 	Let w =width $V = \ell \cdot w$
7.	times		Optional: Area model of multiplication	$V = \ell \cdot w \cdot$
8.	the parcel's height.	Optional: Ruler used to measure height 	Double-headed arrow indicating size of an edge parallel to z-axis 	Let h = height $V = \ell \cdot w \cdot h$

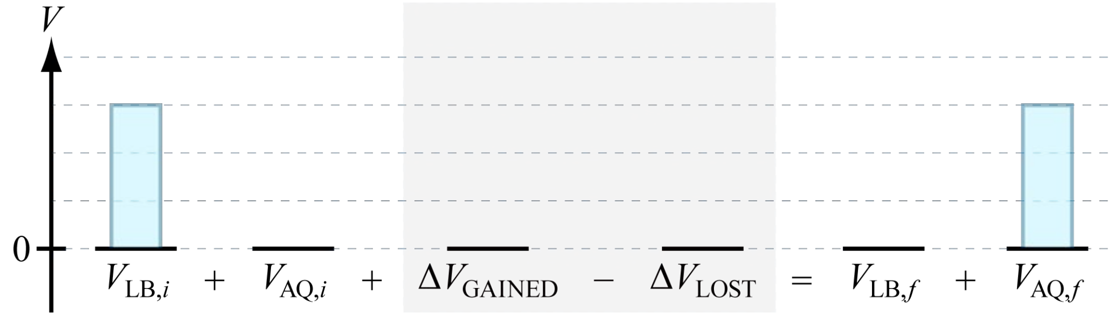
Example 4.1: Sort knowledge about length, width, height, volume, a formula for the volume of a rectangular prism, and an equation for volume conservation.

		Short phrase	Cartoon	Axes and plots	Notation	
1.	**Add** up totals	Incompressible fluids obey **conservation of volume:** change in system's total volume is accounted for by exchange with surroundings.	Water volume in final container exceeds water volume in initial container by amount from faucet. Outline two-containers system.  	Volume bar chart  	$V_{\text{TOT},i} + \Delta V_{\text{GAINED}} - \Delta V_{\text{LOST}} = V_{\text{TOT},f}$ $V_{\text{TOT},i} + \Delta V = V_{\text{TOT},f}$	
					$V_{\text{TOT},i}$	Total initial volume
					ΔV_{GAINED}	Volume gained from surroundings
					ΔV_{LOST}	Volume lost to surroundings
					ΔV	Net volume gained from surroundings
					$V_{\text{TOT},f}$	Total final volume
2.	**Details** formulas	A **rectangular prism's volume** equals the product of the prism's length, width, and height.	Cabinet projection of prism 	Highlight some unit cubes (an object's volume is the number of unit cubes that fit in the object).	$V_{\text{RP}} = \ell w h$	
					ℓ	Prism's length
					w	Prism's width
					h	Prism's height
					V_{RP}	Prism's volume
3.	**Data**	Linear dimensions **length**, **width**, and **height** can be measured using a ruler.	Length, width, and height directly measured using rulers 	 	ℓ	Prism's length
					w	Prism's width
					h	Prism's height
4.		Fluid **volume** is directly measured in a graduated cylinder and indirectly measured by pouring into a graduated cylinder.	Graduated cylinder 	Double-headed arrows label prism's length, width, and height (double-headed arrows indicate endpoint tickmarks of implied coordinate axes).	V	Volume

Add up totals	**Details** formulas	**Data**
Sum of partial amounts equals total. Sum of influences contributes to change. Sum of partial amounts can be changed.	Relate quantities primarily associated with a single object, snapshot, or process.	Physical quantities are values that can be measured using instruments.

Example 5.1: Make a **ADD** chart for the following problem:

A rectangular lunchbox (LB) full of water is emptied completely into an initially empty rectangular aquarium (AQ) that's longer, wider, and taller than the lunchbox. Is the height of the water in the aquarium greater than, less than, or the same as the height of the water initially in the lunchbox?



Add up totals

Details formulas

Data

$$V_{\mathrm{TOT},i} + \Delta V_{\mathrm{GAINED}} - \Delta V_{\mathrm{LOST}} = V_{\mathrm{TOT},f}$$

$$V_{\mathrm{TOT},i} = V_{\mathrm{LB},i} + V_{\mathrm{AQ},i}$$

$$V_{\mathrm{TOT},f} = V_{\mathrm{LB},f} + V_{\mathrm{AQ},f}$$

$$V_{\mathrm{LB},i} = \ell_{\mathrm{LB},i} \cdot w_{\mathrm{LB},i} \cdot h_{\mathrm{LB},i} \qquad V_{\mathrm{AQ},i} = \ell_{\mathrm{AQ},i} \cdot w_{\mathrm{AQ},i} \cdot h_{\mathrm{AQ},i}$$

$$V_{\mathrm{LB},f} = \ell_{\mathrm{LB},f} \cdot w_{\mathrm{LB},f} \cdot h_{\mathrm{LB},f} \qquad V_{\mathrm{AQ},f} = \ell_{\mathrm{AQ},f} \cdot w_{\mathrm{AQ},f} \cdot h_{\mathrm{AQ},f}$$

Example 6.1: Narrate an explanation for Example 5.1.

	Style	Skip statement	Tool	Equivalent parts (of statement & tool)	Populated tool / Point(s) (usually just give point(s) in written explanations)
1.	Pre-write	When asked to write an explanation, you often need not rewrite the problem statement or current line of algebraic work, which is typically already nearby.	Draw a **ADD** chart of all relevant relationships.	Fill in given values in your **ADD** chart. If given partial knowledge (e.g., you're not given a numerical value for ℓ, but you're given that ℓ increases), note the partial knowledge (e.g., draw an up arrow ↑ near ℓ).	Use given information you've just filled in to draw conclusions about quantities in your **ADD** chart.
2.	Default: Medium-long		The volume of water initially in the lunchbox (water's initial product of length, width, and height) converts fully into the volume of the water finally in the aquarium (water's final product of length, width, and height).	The aquarium's greater length and width mean that, at the end, the water at the bottom of the aquarium has greater length and width, respectively, than the length and width of the water initially in the lunchbox.	So, the water has less height in the aquarium.
3.	How to lengthen the medium-long version (typically not advised)		State each equation and name each quantity in Example 5.1's **ADD** chart.	You'd then need to explicitly enumerate each quantity in the **ADD** chart that equals 0.	
4.	Short			The water has a greater length and width when at the bottom of the aquarium than when filling the lunchbox.	So, the water has less height in the aquarium.

Example 7.1: What is $\sqrt{x^2}$? Alice proposes that $\sqrt{x^2} = x$, and Beth propose that $\sqrt{x^2} = |x|$.

	Shiftable input(s)	**Theoretical Expectations**		**Actual results and Measurements**
	x	By Alice's model $\sqrt{x^2} = x$,	By Beth's model $\sqrt{x^2} = \|x\|$,	$\sqrt{x^2}$
1.	-2	-2 😭	2	$\sqrt{(-2)^2} = \sqrt{4} = 2$
2.	-1	-1 😭	1	$\sqrt{(-1)^2} = \sqrt{1} = 1$
3.	0	0	0	$\sqrt{(0)^2} = \sqrt{0} = 0$
4.	1	1	1	$\sqrt{(1)^2} = \sqrt{1} = 1$
5.	2	2	2	$\sqrt{(2)^2} = \sqrt{4} = 2$

In the STEAM table, some of Alice's predictions are **inconsistent** with actual values (when x is negative), so **reject** Alice's model, $\sqrt{x^2} = x$.
In the STEAM table, all of Beth's predictions are **consistent** with actual values, so Beth's model $\sqrt{x^2} = |x|$ **remains in play**.

The STEAM table alone doesn't allow Beth's model to be "ruled in" since the STEAM table doesn't preclude the possibility of additional values of the shiftable input x resulting in values of $\sqrt{x^2}$ that turn out to fail to equal $|x|$.

LESSON 8: Keep track of excluded values and **domain restrictions** using **PAINFUL**.

Example 8.1: Condense $\ln(x) + 2\ln(y)$.

			Forbidden (excluded) values	
			Unstated (implied) exclusions	**Listed (explicit) exclusions**
1.	**Problem** expression, function, or relationship	$\ln(x) + 2\ln(y)$	The argument of a log must be positive, so $x > 0$ and $y > 0$.	(None given by problem statement)
2.	**Algebra STEPs** (possibly showing only **Statements**)	$\ln(x) + \ln(y^2)$	(Even in a sadistic high-school course, you can usually skip this section).	
3.	**Initial Naïve** answer	$\ln(xy^2)$	The argument of a log must be positive, so $xy^2 > 0$. x and y must simultaneously be non-zero (or else $xy^2 = 0$). Requiring $y \neq 0$ guarantees that $y^2 > 0$. So, the only remaining requirement to guarantee $xy^2 > 0$ is the requirement that $x > 0$. Summary: $x > 0, y \neq 0$.	Do the exclusions implied by the naïve answer exclude at least those values that were implicitly excluded by and explicitly excluded for the problem expression, function, or equation? The naïve answer's implied exclusion $x > 0$ correctly reproduces the problem's implied requirement that $x > 0$. No additional statement excluding x-values is needed. However, the naïve answer's implied exclusion $y \neq 0$ allows for the possibility that $y < 0$, which was excluded by the problem's implied requirement that $y > 0$. So, add a statement that $y > 0$.
			If the initial naïve answer is a candidate solution set, plug the candidate solutions back into the original problem relationship. Reject candidate statements that cause the problem relationship to be false.	

	Student response	Score	Expression	Exclusion statement
1.	$\ln(x) + 2\ln(y) = \ln(xy^2), y > 0$	**Full** credit ✓	**Correct**	**Correct** and **efficient**
2.	$\ln(x) + 2\ln(y) = \ln(xy^2), x > 0, y > 0$	Substantial **partial** credit ✗	**Correct**	**Correct**, but **inefficient** (stating $x > 0$ is unnecessary)
3.	$\ln(x) + 2\ln(y) = \ln(xy^2)$	Less **partial** credit ✗	**Correct**	**Incorrect**

For schools that enjoy playful torture, include the phrase "assume all variables are positive" in all problem statements for homework and class examples and never explicitly discuss the **PAINFUL** table above in class. Then, omit the phrase "assume all variables are positive" from exams.

Lesson 1: STEP

Exercise 1.1: Solve for x in each of the following equations.
(a) $-5x + 2 = 12$
(b) $3(x - 1) = 15$
(c) $2x + 5x = 49$
(d) $4x + 5 = 9x$
(e) $x^2 = 9$
(f) $x^4 + 4 = 5x^2$

Exercise 1.2: Explain your reasoning for each step in the STEP tables from Exercise 1.1.

Lesson 2: STEP

Exercise 2.1: If higher frequency for a sound wave means higher pitch for the corresponding sound that's heard, which of frequencies 440 Hz and 880 Hz corresponds to a higher pitch?

Exercise 2.2: Explain your reasoning for Exercise 2.1.

Exercise 2.3: Create a STEP table for the winning argument made by Elle Woods (Reese Witherspoon) in *Legally Blonde* (youtu.be/GSu7BGbyJqc).

Lesson 3: SCAN

Exercise 3.1: Translate: "An object that travels for a duration of time t at constant speed v travels a distance d equal to the product of the duration and speed."

Exercise 3.2: Translate: "The initial value of a quantity plus the change in the value of that quantity equals the quantity's final value. The ratio of the change in price of a basket of goods through a year to the price of that basket of goods at the start of the year is the annual inflation rate measured using that basket."

Lesson 4: ADD-SCAN table

Exercise 4.1: Sort knowledge about ingredient weight in a container, container weight, percentage by weight, total weight in terms of container weights, and total weight of an ingredient in terms of weights of that ingredient in containers using a **ADD-SCAN** table.

Exercise 4.2: Sort knowledge about distance, speed, duration, a distance formula, total journey distance in terms of leg distances, and total journey duration in terms of leg durations using a **ADD-SCAN** table.

Lesson 5: ADD chart

Exercise 5.1: A three-pound bag of trail mix contains 30% raisins by weight. A five-pound bag of trail mix contains 20% raisins by weight. Of the combined trail mix from the two bags, what percent by weight is raisins?

Exercise 5.2: The constant speed with which a boat travels from City A to City B is $5\,\frac{\text{miles}}{\text{hour}}$ less than the constant speed with which the boat travels from City B to City A. The length of the water route between Cities A and B is 50 miles, and the round-trip travel time (not including time spent docked) is 6 hours. What is the boat's speed during each leg of its trip?

Lesson 6: STEP

Exercise 6.1: Narrate your reasoning for exercise 5.1.

Exercise 6.2: Narrate your reasoning for exercise 5.2.

Lesson 7: STEAM

Exercise 7.1: Test the proposal that $\frac{x}{x} = 1$ and another proposed formula of your choosing for $\frac{x}{x}$.

Exercise 7.2: Test the proposal that $\left(\sqrt{x}\right)^2 = x$ and another proposed formula of your choosing for $\left(\sqrt{x}\right)^2$.

Exercise 7.3: Create a STEAM table for the differential diagnosis Mona Lisa Vito (Marisa Tomei) and Vinny Gambini (Joe Pesci) walk through in *My Cousin Vinny* (youtu.be/EvYQTvCVYz0?t=84).

Lesson 8: PAINFUL

Exercise 8.1: Simplify the expression.
(a) $\frac{x^2}{x}$
(b) $\left(\sqrt{2x-1}\right)^2$

Exercise 8.2: Solve for x.
$$\log_{10}(x^2 - 6) = \log_{10}(x)$$

Exercise 8.3: Perform the indicated function operation.
(a) $f(x) = \sqrt{x-2} + 3$
$g(x) = x - \sqrt{x-2} - 3$
Find $f(x) + g(x)$

(b) $f(x) = \frac{x^2-6x-9}{x+2}$
$g(x) = \frac{x-3}{x^2-4}$
Find $\frac{f(x)}{g(x)}$